Chapter 1 Numbers

Activity 1.1
This statement is correct. The number 34.00 has four significant figures, since the end digits of decimal numbers are significant. The number 34 has two significant figures since it has only two non–zero digits. The number 3 400 has two significant figures, since the end zeroes of whole numbers are not significant. The number 0.034 has two significant figures, since the leading zeroes in decimal numbers are not significant.

Exercise 1.1
1. (a) 5 (b) 7 000 (c) 3
2. (a) 1 (b) 2 (c) 3 (d) 3 (e) 3 (f) 1 (g) 3 (h) 3 (i) 4 (j) 6
3. The digits 1 and 9 are significant. The zero digits are not significant. This is because the zeroes at the end of a whole number are not significant.
4. The answer is 0.000108. The number has three significant figures.
5. 1 472 801
   (a) 7 (b) 400 000 (c) (i) Disagree. The number has seven significant figures. (ii) Jabari has not included the zero digit as a significant figure.
6. Digits in bold are significant.
   (a) 10 = 2 s.f. (b) 508 = 3 s.f. (c) 45 000 = 2 s.f. (d) 200 780 = 5 s.f. (e) 1 709 300 = 5 s.f. (f) 9.7 = 2 s.f. (g) 0.01 = 1 s.f. (h) 0.05408 = 4 s.f. (i) 9.670 = 4 s.f. (j) 0.0400 = 3 s.f.

Activity 1.2
1. 4 200 000 2. 4 234 000
3. The second number is more accurate.
4. Cash registers are programmed to round off automatically to the nearest hundredth. It is useful to use rounding off when shopping, to decide approximately how much money is needed.

Rounding off helps to simplify numbers when explaining something to somebody that involves a large number, such as the distance between the earth and the moon.

5. Answers will vary but here are some possible ideas: TB is contagious and spreads through the air. People living with HIV are at much greater risk of becoming sick with TB. A total of 1.77 million people died from TB in 2007. (World Health Organisation) In the 1940s, scientists discovered the first of several drugs now used to treat TB.

Exercise 1.2
1. (a) 346 000 (b) 350 000 (c) 300 000 (d) 346 100 (e) 346 090
2. (a) 0.057 (b) 0.06 (c) 0.0574
3. (a) 14 600 (b) 580 000 (c) 96 420 (d) 1 740 000 (e) 7 300 (f) 1 (g) 0.07 (h) 0.8422
4. The answer is 11 881. Correct to three significant figures, this is 11 900.
5. The answer is 0.0524. Correct to one significant figure, this is 0.05.
6. (a) Monday: 210 Tuesday: 180 Wednesday: 190 Thursday: 72 Friday: 45 (b) 697 students
7. 80 000
8. | Child | Mass on first day of treatment | Mass rounded off to two significant figures | Mass rounded off to three significant figures |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12.89 kg</td>
<td>13 kg</td>
<td>12.9 kg</td>
</tr>
<tr>
<td>B</td>
<td>13.00 kg</td>
<td>13 kg</td>
<td>13.0 kg</td>
</tr>
<tr>
<td>C</td>
<td>25.69 kg</td>
<td>26 kg</td>
<td>25.7 kg</td>
</tr>
<tr>
<td>D</td>
<td>16.943 kg</td>
<td>17 kg</td>
<td>16.9 kg</td>
</tr>
<tr>
<td>E</td>
<td>17.005 kg</td>
<td>17 kg</td>
<td>17.0 kg</td>
</tr>
</tbody>
</table>
9. (a) There are many possible answers, but some correct answers are:
   68 401; 68 429; 68 418
   (b) 68 370; 68 372; 68 373; 68 374

Exercise 1.3
1. (a) 1 500 ÷ 100 = 15
   (b) 43 700 + 900 + 1 400 = 46 000
   (c) 100 × 400 = 40 000
   (d) 28 100 – 14 400 = 13 700
2. (a) 23 100 + 15 500 = 38 600
   (b) 1.03 × 2.6 = 2.678 ≈ 2.68
   (c) 4 860 ÷ 100 = 48.6
   (d) 5 830 – 2 960 = 2 870
3. (a) 123.8 – 56.1 = 67.7
   (b) 3 563.7 + 690.0 = 4 253.7
   (c) 3.5 × 2 = 7
   (d) 2.8 ÷ 1.2 = 2.3
4. 14 800 + 13 100 = 27 900 spectators
5. 4.35 cm × 4.35 cm = 19.0225 cm² ≈ 19.02 cm²
6. 3 750 ÷ 60 = 62.5 = 60 boxes of oranges
7. P4.25 × 20 = P85 = P90
8. Round off the number of people to 80.
   He should buy approximately 160 cups, 80 plates, 160 serviettes and 240 plastic spoons.
9. (a) 3 570 + 12 700 = 16 300
   (b) 3 570 + 12 710 = 16 280
   (c) 3 569.0 + 12 709.0 = 16 278.0
   (d) 16 277.968
   (e) (c) was the most accurate.
   (f) (a) was the least accurate.
10. (a) P134.85 = P130
    P200 – P150 = P70
    She will get approximately P70 change.
    (b) Our estimation would suggest that this is not the correct change. The correct change is in fact P65.15.

Activity 1.3
This is a practical activity to be completed in the classroom.

Exercise 1.4
1. Consider setting up a table as follows:

<table>
<thead>
<tr>
<th>End of day</th>
<th>Money in the piggy bank</th>
<th>Money added to the piggy bank</th>
<th>Money in the piggy bank at the end of the day</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>P1</td>
<td>P1</td>
<td>P2</td>
</tr>
<tr>
<td>2</td>
<td>P2</td>
<td>P2</td>
<td>P4</td>
</tr>
<tr>
<td>3</td>
<td>P4</td>
<td>P4</td>
<td>P8</td>
</tr>
<tr>
<td>4</td>
<td>P8</td>
<td>P8</td>
<td>P16</td>
</tr>
<tr>
<td>5</td>
<td>P16</td>
<td>P16</td>
<td>P32</td>
</tr>
</tbody>
</table>
   (a) There will be P32 in the piggy bank at the end of the fifth day.
   (b) Looking at the pattern in the 4th column of the table, it appears that there is a sequence developing. i.e. 2; 4; 8; 16; 32 ... are all powers of 2: 2₁, 2², 2³, 2⁴, 2⁵
   It follows, therefore, that on the eighth day, the total money in the piggy bank will be 2⁸ = P256
   (c) Since 2¹⁰ = 1 024 and 2⁵ = 512, it appears that by the end of the 10th day, there will be more than P1 000 in the piggy bank.

2. If each of the three free-standing matchsticks are placed at each vertex of the triangle shown, in a vertical position, then brought together to form one vertex at the top, a triangular pyramid is formed. Four triangular faces will therefore result.

3. 

4. The rabbit and the snail are together covering the distance at 9 km per hour (i.e. on adding their speeds). So, they will cover the distance of 81 km in 9 hours. This means in 9 hours they will meet, and the rabbit will have travelled 9 × 8 = 72 km. The snail will have covered 9 km.

   Another option is to use trial and error in a table.

<table>
<thead>
<tr>
<th>Time elapsed</th>
<th>Distance of rabbit from Rabbittown</th>
<th>Distance of snail from Snailville</th>
<th>Total distance covered</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8 km</td>
<td>1 km</td>
<td>9 km</td>
</tr>
<tr>
<td>2</td>
<td>16 km</td>
<td>2 km</td>
<td>18 km</td>
</tr>
<tr>
<td>3</td>
<td>24 km</td>
<td>3 km</td>
<td>27 km</td>
</tr>
<tr>
<td>4</td>
<td>32 km</td>
<td>4 km</td>
<td>36 km</td>
</tr>
<tr>
<td>5</td>
<td>40 km</td>
<td>5 km</td>
<td>45 km</td>
</tr>
<tr>
<td>6</td>
<td>48 km</td>
<td>6 km</td>
<td>54 km</td>
</tr>
<tr>
<td>7</td>
<td>56 km</td>
<td>7 km</td>
<td>63 km</td>
</tr>
<tr>
<td>8</td>
<td>64 km</td>
<td>8 km</td>
<td>72 km</td>
</tr>
<tr>
<td>9</td>
<td>72 km</td>
<td>9 km</td>
<td>81 km</td>
</tr>
<tr>
<td>10</td>
<td>80 km</td>
<td>10 km</td>
<td>90 km</td>
</tr>
<tr>
<td>11</td>
<td>88 km</td>
<td>11 km</td>
<td>99 km</td>
</tr>
</tbody>
</table>

   Using the table, we can see that the total distance of 81 km is covered after 9 hours. At that point, the rabbit and snail meet. The snail will have covered 9 km and the rabbit will have covered 72 km.

5. Since Dikeledi got 10 sweets, she must have won 10 more games than Pako. If Pako won 13 games, Dikeledi must have won 23 games. There were therefore 36 games played in total. Since they played one game each day, Pako was there for 36 days.
6. The triangle is the odd one out as it only has three line segments. The other shapes all have four line segments.

7. L, M, N

This is the sequence of consonants.

8. $7 \times 7 \times 7 \times 7 \times 7 = 16,807$ measures of flour

9. 1991 and 2112

**Extension activity**

These are some possible answers:

$1 + 2 + 34 - 5 + 67 - 8 + 9 = 100$

$12 + 3 - 4 + 5 + 6 + 78 - 9 = 100$

$123 + 45 - 6 + 7 + 89 = 100$

$123 - 45 + 67 - 8 + 90 = 100$

In simpler terms, the rule could be stated as:

- add 1 to the term number
- multiply the sum by the term number
- divide the product by 2.

**Exercise 1.5**

1. (a) 10, 12, 14  
   (b) 35, 40, 45  
   (c) −9, −11, −13  
   (d) $\frac{1}{2}, \frac{13}{4}, 2$

2. (a) $T_n = -3n + 5$  
   (b) $T_{100} = -295$

3. (a) $T_n = 3n + 1$  
   (b) $T_7 = 22$

4. (a) $T_n = 3n - 13$  
   (b) $T_{12^{th}}$ term  
   (c) $T_{25} = 37$

**Exercise 1.6**

1. (a) 7.5%, 9%, 10.5%  
   (b) $\frac{1}{2}, \frac{13}{4}, 2$

2. (a) 7, 10, 13, 16, 19

3. (a) $T_n = -3n + 5$  
   (b) $T_{100} = -295$

4. (a) $-3, -1, 1$  
   (b) $T_n = 2n - 13$

5. (a) $T_n = 3n + 1$  
   (b) $T_7 = 22$

**Revision exercise**

1. (a) 1  
   (b) 4  
   (c) 4  
   (d) 1  
   (e) 4  
   (f) 2

2. (a) (i) 3 800  
   (ii) 3 780  
   (b) (i) 15 000  
   (ii) 15 050  
   (c) (i) 490  
   (ii) 490  
   (d) (i) 160 000  
   (ii) 163 970  
   (e) (i) 9 200  
   (ii) 9 210  
   (f) (i) 1 800 000  
   (ii) 1 784 540

3. (a) (i) 0.785  
   (ii) 0.78  
   (b) (i) 8.65  
   (ii) 8.65  
   (c) (i) 0.344  
   (ii) 0.34  
   (d) (i) 164  
   (ii) 163.98

4. (a) $330 \times 190 = 62,700 = 63,000$  
   (b) $4600 \div 450 = 10.222\ldots = 10$  
   (c) $0.067 + 0.1 = 0.167 = 0.17$  
   (d) $45 - 35 = 10$

5. The smallest tap fills $\frac{1}{20}$ of the barrel in 1 minute.  
   The middle tap fills $\frac{1}{12}$ of the barrel in 1 minute.  
   The largest tap fills $\frac{1}{5}$ of the barrel in 1 minute.  
   Together they fill $\frac{1}{20} + \frac{1}{12} + \frac{1}{5} = \frac{1}{3}$ of the barrel in 1 minute. Therefore, the whole barrel is filled in 3 minutes.
6. (a) 73, 71, 69   (b) 1.5, 1.75, 2
   (c) 41%, 48%, 55%   (d) -42, -52, -62
   (e) 8, 13, 21 (Fibonacci sequence)
   (f) $\frac{16}{10}, \frac{21}{10}, \frac{26}{10}$

7. 2, 5, 8, 11, 14

8. (a) $T_n = 3n$   (b) $T_n = -2n + 1$
Chapter 2 Squares, square roots, cubes and cube roots

Exercise 2.1

1. Side length of square (cm) | Area of square (cm²)
---|---
(a) 6 | $6 \times 6 = 36$
(b) 7 | $7 \times 7 = 49$
(c) 8 | $8 \times 8 = 64$
(d) 9 | $9 \times 9 = 81$
(e) 10 | $10 \times 10 = 100$
(f) 11 | $11 \times 11 = 121$

2. (a) $3^2$ | (b) $9^2$ | (c) $\left(\frac{1}{2}\right)^2$ | (d) $\left(\frac{1}{8}\right)^2$

3. (a) $9$ | (b) $1$ | (c) $16$ | (d) $169$
(e) $\frac{1}{4}$ | (f) $\frac{1}{9}$ | (g) $225$
(h) 10 000 | (i) $\frac{16}{25}$ | (j) $\frac{49}{100}$ | (k) 0.16
(l) 0.0001 | (m) $\frac{25}{81}$ | (n) 0.04 | (o) $\frac{16}{49}$

4. (a) $25$ | (b) $100$ | (c) $81$ | (d) $400$
(e) $\frac{1}{64}$ | (f) $0.49$ | (g) $1225$ | (h) 0.09
(i) $\frac{4}{25}$ | (j) $\frac{81}{100}$

5. (a) 14.44 | (b) 144.400
(c) 0.1444 | (d) 0.001444

6. Surface area of floor is $11 \times 11 = 121$ m²
$121 \text{ m}^2 \div 1 \text{ m}^2 = 121$
There are 121 tiles needed.

7. (a) $4 + 9 = 13$ | (b) $49 + 16 = 65$
(c) $\frac{1}{4} + \frac{1}{16} = \frac{5}{16}$ | (d) $0.01 + 0.36 = 0.37$
(e) $\frac{1}{64} + \frac{9}{64} = \frac{10}{64}$ | (f) $\frac{25}{36} + \frac{9}{36} = \frac{34}{36}$

8. (a) $n = 2$ | (b) $n = 25$ | (c) $n = 10$

Exercise 2.2

1. (a) 2 | (b) 5 | (c) 7 | (d) 10
(e) 6 | (f) $\frac{1}{2}$ | (g) $\frac{1}{8}$ | (h) 0.1
(i) $\frac{3}{5}$ | (j) $\frac{2}{3}$

2. 12 cm

3. (a) 1 | (b) 2 | (c) 9 | (d) 4
(e) 14 | (f) 15 | (g) $\frac{1}{5}$
(h) $\frac{4}{3}$ or $1\frac{1}{3}$ | (i) $\frac{3}{2}$ | (j) 0.3

4. 5 m

5. (a) $225 = 5 \times 45$
$= 5 \times 5 \times 9$
$= 5^2 \times 3^2$
$\sqrt{225} = 5 \times 3 = 15$

(b) $256 = 2 \times 128$
$= 2 \times 2 \times 64$
$= 2 \times 2 \times 2^6$
$\sqrt{256} = 2 \times 2 \times 2 = 16$

(c) $36 = 2 \times 18$
$= 2 \times 2 \times 9$
$= 2 \times 2 \times 3 \times 3$
$= 2^2 \times 3^2$
$\sqrt{36} = 2 \times 3 = 6$

(d) $400 = 2 \times 200$
$= 2 \times 2 \times 100$
$= 2 \times 2 \times 10 \times 10$
$= 2 \times 2 \times 2 \times 5 \times 2 \times 5$
$= 2^3 \times 5^2$
$\sqrt{400} = 2 \times 2 \times 5 = 20$

(e) $1764 = 2 \times 882$
$= 2 \times 2 \times 441$
$= 2 \times 2 \times 3 \times 147$
$= 2 \times 2 \times 3 \times 3 \times 49$
$= 2 \times 2 \times 3 \times 3 \times 7 \times 7$
$= 2^3 \times 3^2 \times 7^2$
$\sqrt{1764} = 2 \times 3 \times 7 = 42$

6. 9

7. (a) $n = 13$ | (b) $n = \frac{1}{8}$ | (c) $n = 100$
(d) $n = \frac{1}{4}$ | (e) $n = 16$ | (f) $n = 3$

Activity 2.1

<table>
<thead>
<tr>
<th>3</th>
<th>6</th>
<th>$\frac{1}{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{3}$</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>$\frac{1}{2}$</td>
<td>9</td>
</tr>
</tbody>
</table>

Exercise 2.3

1. (a) 8 | (b) 1 | (c) 27 | (d) 125
(e) 1000 | (f) 216 | (g) 729 | (h) 64
(i) $\frac{1}{8}$ | (j) $\frac{8}{27}$

2. (a) 64 | (b) 125 | (c) $\frac{1}{64}$ | (d) $\frac{1}{125}$
(e) 512 | (f) 0.001 | (g) 0.027 | (h) $\frac{8}{27}$
(i) $\frac{125}{343}$ | (j) $\frac{27}{125}$
Exercise 2.5

1. (a) 50  (b) 63  (c) 54  (d) 124
   (e) 418  (f) 0.875  (g) 0.258
   (h) 0.8 (2 d.p.)  (i) 0.47 (2 d.p.)
   (j) 2.4 (2 d.p.)  (k) 2.6 (2 d.p.)
   (l) 0.77 (2 d.p.)  (m) 2.39 (2 d.p.)
   (n) 3.63 (2 d.p.)  (o) 0.02 (2 d.p.)

2. (a) 1 089  (b) 27 225  (c) 151 321
   (d) 164 025  (e) 1 000 000
   (f) 0.02 (2 d.p.)  (g) 0.49
   (h) 15 (2 d.p.)  (i) 0 (2 d.p.)
   (j) 34.03 (2 d.p.)  (k) 745.29
   (l) 53.78 (2 d.p.)  (m) 0.26 (2 d.p.)
   (n) 61.15 (2 d.p.)  (o) 0.25

3. (a) 11  (b) 15  (c) –20  (d) 25
   (e) 63  (f) –0.3  (g) 0.75  (h) 1.2
   (i) 50.5 (j) 0.96 (2 d.p.)
   (k) 0.93 (l) 1.09 (2 d.p.)
   (m) 1.84 (2 d.p.)  (n) 1.73 (2 d.p.)
   (o) –0.25

4. (a) 1 728  (b) 3 375  (c) –12 167
   (d) 1 030 301  (e) 125 000 000
   (f) 0.216  (g) 0 (2 d.p.)
   (h) 190.11 (2 d.p.) (i) 0.09 (2 d.p.)
   (j) 0.027  (k) 386.74 (2 d.p.)
   (l) 4.34 (2 d.p.)  (m) 12.07 (2 d.p.)
   (n) –0.512  (o) –47.44 (2 d.p.)

5. 662 cm²

6. 12.8 cm × 12.8 cm = 163.84 cm²
   10.4 cm × 10.4 cm = 108.16 cm²
   163.84 cm² – 108.16 cm² = 55.68 cm²
   The area of the picture frame is 55.68 cm².

Exercise 2.4

1. (a) 2  (b) 10  (c) 4  (d) 3
   (e) 5  (f) 1/4  (g) 1/6  (h) 2/3
   (i) 0.2  (j) 0.5

2. 

<table>
<thead>
<tr>
<th>Volume of cube (cm³)</th>
<th>27</th>
<th>64</th>
<th>729</th>
<th>1 000</th>
<th>1 331</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side length of cube (cm)</td>
<td>3</td>
<td>4</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

3. 5 m × 5 m × 5 m

4. (a) 5 + 6 = 11  (b) 8 – 4 = 4
   (c) 2 × 10 = 20  (d) 9 ÷ 3 = 3
   (e) 3 + 5 = 8  (f) 1/4 × 1/3 = 1/12
   (g) 2/3 ÷ 2/3 = 1

5. 13 824 = 2³ × 2³ × 2³ × 3³
   \sqrt[3]{13 824} = 2 × 2 × 2 × 3 = 24

6. \sqrt[3]{\frac{27}{8}} = \frac{\sqrt[3]{27}}{\sqrt[3]{8}} = \frac{3}{2} = 1 \frac{1}{2}

7. (a) 1 331 = 11 × 121
   = 11 × 11 × 11
   \sqrt[3]{1 331} = 11
   (b) 2 197 = 13 × 169
   = 13 × 13 × 13
   \sqrt[3]{2 197} = 13
   (c) 2 744 = 2 × 1 372
   = 2 × 2 × 686
   = 2 × 2 × 2 × 343
   = 2 × 2 × 2 × 7 × 49
   = 2 × 2 × 2 × 7 × 7 × 7
   \sqrt[3]{2 744} = 2 × 2 × 7 = 28

Exercise 2.2

04/04/2016; 05/05/2025; 06/06/2036; 07/07/2049;
08/08/2064; 09/09/2081

Revision exercise

1. (a) 16  (b) 49  (c) 144  (d) 1/16
   (e) \frac{81}{100}  (f) 8  (g) 125  (h) 216
   (i) 0.001  (j) \frac{27}{8}

2. (a) 7  (b) 10  (c) 8  (d) 9
   (e) \frac{1}{2}  (f) \frac{1}{4}  (g) \frac{3}{5}  (h) 32
   (i) 28  (j) 51
3.  (a) 3   (b) 2   (c) 10   (d) $\frac{1}{4}$
(e) 0.2
(f) $1\ 728 = 2 \times 864$
   = $2 \times 2 \times 432$
   = $2 \times 2 \times 2 \times 216$
   = $2 \times 2 \times 2 \times 2 \times 108$
   = $2 \times 2 \times 2 \times 2 \times 3 \times 36$
   = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 12$
   = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 6$
   = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 2 \times 3$
   = $2^3 \times 2^3 \times 3^3$

$\sqrt[3]{1\ 728} = 2 \times 2 \times 3 = 12$

(g) $3\ 375 = 3 \times 1\ 125$
   = $3 \times 3 \times 375$
   = $3 \times 3 \times 3 \times 125$
   = $3 \times 3 \times 3 \times 5 \times 5 \times 5$
   = $3^3 \times 5^3$

$\sqrt[3]{3\ 375} = 3 \times 5 = 15$

4.  (a) 2 209   (b) 0.2 (2 d.p.)
(c) 12.17 (2 d.p.)   (d) 4 096
(e) 1.42 (2 d.p.)   (f) 84
(g) 1.95   (h) 0.45 (2 d.p.)
(i) 16   (j) 0.61 (2 d.p.)

5.  Side length of vegetable patch is 16 m. Perimeter is $16 \times 4 = 64$ m.
Fencing required is 64 m.

6.  2.5 m
Chapter 3 Indices

Exercise 3.1
1. (a) $4^3$ (b) $2^3$ (c) $10^2$ (d) $9^4$
   (e) $8^6$ (f) $5^3$ (g) $12^3$ (h) $7^7$
   (i) $m^7$
2. (a) $3 \times 3 \times 3 \times 3$ (b) $8 \times 8$
   (c) $9 \times 9 \times 9$
   (d) $10 \times 10 \times 10 \times 10 \times 10$
   (e) $11 \times 11 \times 11 \times 11 \times 11$
   (f) $12 \times 12 \times 12 \times 12$
   (g) $137 \times 137 \times 137$
   (h) $4 \times 4 \times 4 \times 4 \times 4$
   (i) $7 \times 7 \times 7 \times 7 \times 7 \times 7$
   (j) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
3. (a) $t^5$ (b) $c^d d^e$ (c) $j^y$ (d) $kp^q r^t$
   (e) $u^r v^r$ (f) $4y^r$
4. (a) $r \times r \times r \times r \times r$
   (b) $e \times e \times e$
   (c) $t \times t \times m \times m$
   (d) $h \times w \times w \times w \times w$
   (e) $5 \times f \times f \times f \times y \times y \times y$
   (f) $u \times u \times u \times u \times n \times n \times n \times b \times b \times b \times b \times b$
5. (a) $10^6$ (b) $6$ (c) $10$ and $6$
   (d) $10 \times 10 \times 10 \times 10 \times 10 \times 10$
6. $16 = 2^4$ and $16 = 4^2$
7. Challenge:
   (a) $8 \times 8 \times 8 \times 8 \times 8$ (b) $8^5 = 32768$

Exercise 3.2
1. There may be different, correct answers to this question, since the students have not yet been shown how to use prime factors.
   (a) $10^2$ (b) $9^2$ (c) $4^2 \times 3$ (d) $5 \times 2^3$
   (e) $3 \times 7^2$ (f) $5^2 \times 7^2$ (g) $2^2 \times 10^2$
   (h) $2^2 \times 12^2$ (i) $5^4$ (j) $10^3$
2. (a) $128 = 2^7$ (b) $256 = 2^8$
   (c) $1280 = 2^8 \times 5$ (d) $12800 = 2^9 \times 5$
   (e) $2^7 \times 5$
3. $2 \times 3 \times 5^3$ or $6 \times 5^3$

Exercise 3.3
1. (a) no (b) yes (c) yes (d) no
   (e) yes (f) no (g) no (h) no
   (i) yes (j) yes
2. (a) 1, 2, 3, 4, 6, 12
   (b) 1, 2, 4, 5, 10, 20
   (c) 1, 5, 7, 35
   (d) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
   (e) 1, 2, 4, 5, 8, 10, 16, 20, 40, 80
   (f) 1, 3, 5, 7, 15, 21, 35, 105
   (g) 1, 2, 3, 10, 13, 26, 65, 130
   (h) 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160
   (i) 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200
   (j) 1, 2, 5, 10, 25, 50, 125, 250
Activity 3.1
Consider the first 60 prime numbers:

\[
\begin{array}{c|c}
2 & 3 \\
3 & 5 \\
5 & 7 \\
7 & 11 \\
11 & 13 \\
13 & 17 \\
17 & 19 \\
19 & 23 \\
23 & 29 \\
29 & 31 \\
31 & 37 \\
37 & 41 \\
41 & 43 \\
43 & 47 \\
47 & 53 \\
53 & 59 \\
59 & 61 \\
61 & 67 \\
67 & 71 \\
71 & 73 \\
73 & 79 \\
79 & 83 \\
83 & 89 \\
89 & 97 \\
97 & 101 \\
101 & 103 \\
103 & 107 \\
107 & 109 \\
109 & 113 \\
113 & 127 \\
127 & 131 \\
131 & 137 \\
137 & 139 \\
139 & 149 \\
149 & 151 \\
151 & 157 \\
157 & 163 \\
163 & 167 \\
167 & 173 \\
173 & 179 \\
179 & 181 \\
181 & 191 \\
191 & 193 \\
193 & 197 \\
197 & 199 \\
199 & 211 \\
211 & 223 \\
223 & 227 \\
227 & 229 \\
229 & 233 \\
233 & 239 \\
239 & 241 \\
241 & 251 \\
251 & 257 \\
257 & 263 \\
263 & 269 \\
269 & 271 \\
271 & 277 \\
277 & 281 \\
\end{array}
\]

Students test any 10 prime numbers. They could set their work out as follows:

\[
\begin{align*}
13 & \times 13 - 1 = 168 \quad (168 \div 24 = 7) \\
29 & \times 29 - 1 = 840 \quad (840 \div 24 = 35)
\end{align*}
\]

Activity 3.2
3. (a) \(2 \times 2 \times 2 \times 2 \times 2 = 2^5\)
(b) \(3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7\)
(c) \(4 \times 4 \times 4 \times 4 \times 4 = 4^6\)
(d) \(x \times x \times x \times x \times x \times x = x^6\)
(e) \(p \times p \times p \times p \times p \times p = p^6\)
(f) \(r \times r \times r \times r \times r \times r = r^6\)

4. It appears as though we could add the indices of the numbers being multiplied. The sum is the index of the product (answer). This would be a much shorter method than using repeated multiplication.

Exercise 3.4
1. (a) \(5 \times 5 \times 5 \times 5 \times 5 = 5^5\)
(b) \(3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6\)
(c) \(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8\)
(d) \(7 \times 7 \times 7 \times 7 \times 7 \times 7 = 7^6\)
(e) \(6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^6\)
(f) \(a \times a \times a \times a \times a \times a = a^6\)

2. (a) \(2^{11}\)  (b) \(5^6\)  (c) \(9^{10}\)  (d) \(8^{12}\)
(e) \(10^9\)  (f) \(y^{14}\)

3. (a) \(2^2 + 2^3 = 4 + 8 = 12\). But \(12 \neq 2^5\)
(b) \(3^2 + 3^4 = 9 + 81 = 90\). But \(90 \neq 3^6\)
(c) \(10^4 + 10^2 = 10000 + 100 = 10100\). But \(10 \times 100 \neq 10^6\)

We can conclude that the law of multiplying powers does not apply for addition of powers with the same base.
Activity 3.3
3. (a) \(2^3 \times 2^3 \times 2^2 \times 2 = 2^9\)
(b) \(3^3 \times 3^3 \times 3^3 \times 3 = 3^9\)
(c) \(\frac{7^3}{7} = 7\)
(d) \(8^3 \times 8 \times 8 = 8^2\)
(e) \(\frac{x^3 y^4 y^3 y^3 y^3 y^3}{x^3 y^3 y^3 y^4} = y^6\)
(f) \(\frac{x^5 y^4 x^5 y^4 x^5 y^4}{x^5 y^4 x^5 y^4 x^5 y^4} = x^2\)
4. It appears as though we could subtract the indices of the numbers being divided. The difference is the index of the quotient (answer). This would be a much shorter method than using repeated multiplication and cancelling.

Exercise 3.5
1. (a) \(\sqrt[3]{5^3} = 5\)
(b) \(\sqrt[3]{6^3} \times \sqrt[3]{6} = 6\)
(c) \(\sqrt[3]{3^3 \times 3 \times 3 \times 3} = 3^4\)
(d) \(\sqrt[3]{4^3 \times 4 \times 4 \times 4} = 4^3\)
(e) \(\sqrt[3]{x^3 y^3 z^3 t^3 t^3 t^3} = t^3\)
(f) \(\sqrt[3]{k^3 k^3 k^3 k^3 k^3} = k\)
2. (a) \(9^3\) (b) \(4^3\) (c) \(8^6\)
(d) \(12^4\) (e) \(10^6\) (f) \(8^{11}\)
3. (a) \(6^3 - 6^2 = 7 776 - 36 = 7 740\), but \(7 740 \neq 6\)
(b) \(3^7 - 3^2 = 2 187 - 9 = 2 178\), but \(2 178 \neq 3\)
(c) \(2^4 - 2^3 = 16 - 8 = 8\), but \(8 \neq 2\)

Activity 3.4
3. (a) \(5^3 \times 5^3 \times 5^3\)
   \(= (5 \times 5 \times 5) \times (5 \times 5 \times 5) \times (5 \times 5 \times 5)\)
   \(= 5^9\)
(b) \(7^4 \times 7^4 \times 7^4\)
   \(= (7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7) \times (7 \times 7 \times 7 \times 7)\)
   \(= 7^{12}\)
(c) \(10^2 \times 10^2 \times 10^2 \times 10^2\)
   \(= (10 \times 10) \times (10 \times 10) \times (10 \times 10) \times (10 \times 10)\)
   \(= 10^8\)
(d) \(e^3 \times e^3 \times e^3\)
   \(= (e \times e \times e \times e \times e \times e) \times (e \times e \times e \times e \times e \times e) \times (e \times e \times e \times e \times e \times e)\)
   \(= e^{15}\)
(e) \(x^3 y^3 x^3 y^3 x^3 y^3\)
   \(= (x \times x \times x) \times (x \times x \times y) \times (x \times x \times y) \times (x \times x \times y) \times (x \times x \times y) \times (x \times x \times y)\)
   \(= x^{15} y^9\)
(f) \(5h^2 r^2 \times 5h^2 r^2 \times 5h^2 r^2 \times 5h^2 r^2\)
   \(= (5 \times h \times h \times h \times r \times r) \times (5 \times h \times h \times h \times r \times r) \times (5 \times h \times h \times h \times r \times r) \times (5 \times h \times h \times h \times r \times r)\)
   \(= 5^4 h^{12} r^{12}\)
4. Multiply the exponent of the number or letter within the brackets by the exponent outside the brackets.

Exercise 3.6
1. (a) \(5^8\) (b) \(3^{12}\) (c) \(2^{16}\)
   (d) \(f^8\) (e) \(b^{31}\) (f) \(3^{50} g^{72} h^p\)
2. (a) \(3^{10}\) (b) \(7^{21}\) (c) \(v^{10}\)
   (d) \(4^{11}\) (e) \(2^{4^{21}}\) (f) \(5^{4^{w^{x^{15}}}}\)

Activity 3.5
2. (a) \(\frac{\sqrt[3]{x^3 y^3 z^3 t^3 t^3 t^3}}{c} \times c \times c \times c \times c \times c = \frac{1}{c}\)
   but \(\frac{c^6}{c^3} = c^3\)
(b) \(\frac{\sqrt[3]{x^3 y^3 z^3 t^3 t^3 t^3}}{c} \times c \times c \times c \times c \times c = \frac{1}{c}\)
   But \(\frac{c^6}{c^3} = c^3\)
(c) \(\frac{\sqrt[3]{x^3 y^3 z^3 t^3 t^3 t^3}}{c} \times c \times c \times c \times c \times c = \frac{1}{c} \times \frac{1}{c}\)
   But \(\frac{c^6}{c^3} = c^3\)

Activity 3.6
2. (a) \(\frac{\sqrt[3]{x^3 y^3 z^3 t^3 t^3 t^3}}{c} \times c \times c \times c \times c \times c = \frac{1}{c}\)
   But \(\frac{c^6}{c^3} = c^3\)
(b) \(\frac{\sqrt[3]{x^3 y^3 z^3 t^3 t^3 t^3}}{c} \times c \times c \times c \times c \times c = \frac{1}{c}\)
   But \(\frac{c^6}{c^3} = c^3\)
(c) \(\frac{\sqrt[3]{x^3 y^3 z^3 t^3 t^3 t^3}}{c} \times c \times c \times c \times c \times c = \frac{1}{c} \times \frac{1}{c}\)
   But \(\frac{c^6}{c^3} = c^3\)

Activity 3.7
3. \(\left(\frac{y}{x}\right)^{\frac{1}{3}}\)
   \(= \frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}\)
   \(= y^{\frac{1}{3}}x^{-\frac{1}{3}}\)
   When we raise a power to a power, we multiply the exponents. So, if one power is the inverse of the other power, the product will be 1. In other words, we will simply be left with the base of the original problem.

Exercise 3.7
1. (a) 1 (b) 1 (c) 1 (d) 1
   (e) 1 (f) 2 (g) 9 (h) 4
   (i) 12 (j) b
2. (a) \( \frac{1}{6^2} \)  
(b) \( \frac{1}{8^3} \)  
(c) \( \frac{1}{p^6} \)  
(d) \( \frac{7^2}{9} \)  
(e) \( \frac{2}{5^2} \)  
(f) \( 2^6 \)  
(g) \( \frac{1}{12^1} \)  
(h) \( \frac{1}{7} + 1 = 1\frac{1}{7} \)  
(i) \( 8k^8 \)  
(j) \( \frac{a^7}{b^5} \)

Extension activity

<table>
<thead>
<tr>
<th>A</th>
<th>( a^2 )</th>
<th>H</th>
<th>( a^{12} )</th>
<th>O</th>
<th>( 5a^2 )</th>
<th>V</th>
<th>( \frac{5}{a^2} )</th>
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<td>( \frac{1}{a} )</td>
<td>I</td>
<td>( 5a )</td>
<td>P</td>
<td>( a^2 )</td>
<td>W</td>
<td>1</td>
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<tr>
<td>C</td>
<td>( a^{20} )</td>
<td>J</td>
<td>( \frac{1}{4} ) or ( \frac{1}{2} )</td>
<td>Q</td>
<td>4</td>
<td>X</td>
<td>( 2a^6 )</td>
</tr>
<tr>
<td>D</td>
<td>( a^4 ) or ( a^2 )</td>
<td>K</td>
<td>( a^{20} )</td>
<td>R</td>
<td>( a^{14} )</td>
<td>Y</td>
<td>( a^2 )</td>
</tr>
<tr>
<td>E</td>
<td>( \frac{1}{a} )</td>
<td>L</td>
<td>( \frac{1}{5} )</td>
<td>S</td>
<td>( \frac{5}{a^2} )</td>
<td>Z</td>
<td>( (-3)^3a^6 )</td>
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<tr>
<td>F</td>
<td>( \frac{1}{a^3} )</td>
<td>M</td>
<td>( 2a^5 )</td>
<td>T</td>
<td>( \frac{1}{a^6} )</td>
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</tr>
<tr>
<td>G</td>
<td>( 3^3a^{13} )</td>
<td>N</td>
<td>2a</td>
<td>U</td>
<td>( 24a^{14} )</td>
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</tr>
</tbody>
</table>

1. BOXING  
2. HOCKEY  
3. TENNIS  
4. SOCCER

Exercise 3.8

1. (a) Population at time 0 = \( 50 \times 2^0 = 50 \)  
   Population at time 1 = \( 50 \times 2^1 = 100 \)  
   Population at time 2 = \( 50 \times 2^2 = 200 \)  
   Population at time 3 = \( 50 \times 2^3 = 400 \)  
   Population at time 8 = \( 50 \times 2^8 = 12 800 \)  
(b) \( t = 50 \times 2^t \)  

2. (a) \( 2^1, 2^2, 2^3, 2^4 \)  
   (b) \( 2^3 = 32 \) orange beads  

3. (a) \( a^9b^2 \)  
   (b) \( 1 \)  
   (c) \( \frac{6a^2b}{5} \)  
   (d) \( a^{19} \)  
   (e) \( 12a^4 \)  
   (f) \( \frac{4a^4}{3b^6c^3} \)  

4. \( 5^4 \) fleas  
5. \( a = 10 \)  
6. \( 2 \times 3^4 \) trees

Exercise 3.9

1. (a) \( 5 000 \)  
   (b) \( 78 000 \)  
   (c) \( 0.002 \)  
   (d) \( 0.00045 \)  
   (e) \( 287 000 \)  
   (f) \( 0.31 \)  
   (g) \( 990 000 000 \)  
   (h) \( 24 500 \)  
   (i) \( 0.0000059 \)  
   (j) \( 8 640 \)

2. (a) \( a = 3 \)  
   (b) \( a = 4 \)  
   (c) \( a = 5 \)  
   (d) \( a = 6 \)  
   (e) \( a = -2 \)  
   (f) \( a = 1 \)  
   (g) \( a = -3 \)  
   (h) \( a = 3 \)  
   (i) \( a = 4 \)  

3. (a) (ii)  
   (b) (iii)  
   (c) (i)  
   (d) (i)  
   (e) (ii)

4. (a) \( 3 \times 10^4 \)  
   (b) \( 1.4 \times 10^4 \)  
   (c) \( 5 \times 10^4 \)  
   (d) \( 1.5 \times 10^5 \)  
   (e) \( 1 \times 10^6 \)  
   (f) \( 6.75 \times 10^2 \)  
   (g) \( 1.28 \times 10^4 \)  
   (h) \( 5 \times 10^{-2} \)  
   (i) \( 6 \times 10^{-3} \)  
   (j) \( 7 \times 10^{-4} \)  
   (k) \( 6.43 \times 10^{-3} \)  
   (l) \( 1.23 \times 10 \)  
   (m) \( 1.508 \times 10^2 \)  
   (n) \( 1.308785 \times 10^4 \)  

5. (a) Dinosaur: \( 1 \times 10^4 \) kg  
   (b) Car: \( 1 \times 10^3 \) kg  
   (c) Asian elephant: \( 5 \) 000 kg  
   (d) Blue whale: \( 1.7 \times 10^5 \) kg  
   (e) Building: \( 7 \times 300 \) 000 kg

Extension activity

<table>
<thead>
<tr>
<th>Index</th>
<th>Base 2</th>
<th>Base 3</th>
<th>Base 4</th>
<th>Base 5</th>
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<tbody>
<tr>
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<td>4 096</td>
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<td>65 536</td>
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<td>19 683</td>
<td>262 144</td>
<td>1 953 125</td>
</tr>
<tr>
<td>10</td>
<td>1 024</td>
<td>59 049</td>
<td>1 048 576</td>
<td>9 765 625</td>
</tr>
</tbody>
</table>

(a) \( 2, 4, 6 \) or \( 8 \)  
(b) \( 1, 3, 7 \) or \( 9 \)  
(c) Base 4: Only the digits 4 and 6 are seen in the units places.  
Base 5: Only the digit 5 is seen in the units places.

Exercise 3.10

1. (a) \( 2 \times 10^{-9} \) g/ml  
   (b) \( 1.2 \times 10^{-3} \) g/ml  
   (c) \( 0.000005 \) g/ml

2. \( 300 \ 000 \ 000 \) m/s
3. (a) \(7 \times 10^{-6}\) g  
(b) \(0.0007 = 7 \times 10^{-4}\) g  
(c) \(0.0021 = 2.1 \times 10^{-3}\) g  
4. \(1020000 = 1.02 \times 10^6\) Smarties  
5. (a) \(5.9 \times 10^6\) tons  
(b) \(2.565\) tons  
(c) \(2565\) kg = \(2.565 \times 10^3\) kg  

Revised exercise  
1. (a) \(2 \times 2 \times 2 \times 2\)  
(b) \(6 \times 6 \times 6\)  
(c) \(5 \times 5 \times 5 \times 5 \times 5\)  
(d) \(2 \times 2 \times 2 \times 2 \times 2\)  
(e) \(3 \times 3\)  
(f) \(4 \times 4 \times 4 \times p \times p \times p \times p \times p\)  
2. (a) \(3^4\)  
(b) \(2^5\)  
(c) \(10^3\)  
(d) \(8^3\)  
(e) \(y^5\)  
(f) \(5^3c^3\)  
3. (a) \(6^2\)  
(b) \(2 \times 5^2\)  
(c) \(13 \times 5\)  
(d) \(2 \times 6^2\)  
(e) \(2 \times 3^2 \times a\)  
4. (a) \(2^3 \times 5\)  
(b) \(2 \times 3 \times 5\)  
(c) \(2^2 \times 5^2\)  
(d) \(2^3 \times 3 \times 5\)  
(e) \(2 \times 5^3\)  
5. (a) The law of multiplying powers  
(b) The rule of zero indices  
(c) The law of dividing powers  
(d) The rule of fractional indices  
(e) The rule of negative indices  
(f) The law of raising a power to a power  
6. (a) \(7^{12}\)  
(b) \(3\)  
(c) \(\frac{1}{b^{11}}\)  
(d) \(3^7 u^5\)  
(e) \(4^4\)  
7. (a) \(4 \times 10^3\)  
(b) \(1.25 \times 10^6\)  
(c) \(4.98 \times 10^2\)  
(d) \(8 \times 10^{-2}\)  
(e) \(6.5 \times 10^{-4}\)  
8. (a) \(9.79 \times 10^2\) m  
(b) \(8.9 \times 10^3\) m  
(c) \(2.72 \times 10^6\) m  
(d) \(8.18 \times 10^2\) m  
(e) \(6 \times 10^6\) m  
From tallest to shortest:  
\(8.9 \times 10^3\) m, \(9.79 \times 10^2\) m, \(8.18 \times 10^2\) m,  
\(6 \times 10^6\) m, \(2.72 \times 10^6\) m.
Chapter 4 Money

Exercise 4.1
1. (a) (i) P7 767.50 (ii) P11 595.00 (iii) P12 117.00
(b) Rent: P9 600.00  (c) Shoes and beads
(d) Overheads
(e) Since her rent is most costly, perhaps she could consider moving premises.
She could look at reducing her telephone bill.
She could investigate getting her shoes from a different source, thereby reducing the cost of the shoes.

2. (a)

<table>
<thead>
<tr>
<th>Type of cost</th>
<th>Calculation</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour cost</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Farm labourers</td>
<td>Cost per person 200 hours × P15.50/h = P3 100.00 – Cost: 5 × P3 100 = P15 500.00</td>
<td></td>
</tr>
<tr>
<td>(b) Receptionist</td>
<td>Cost: 160 hours × P15.45 = P2 472.00</td>
<td></td>
</tr>
<tr>
<td>(c) Accountant</td>
<td>Cost: 40 hours × P70.70 = P2 828.00</td>
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</tr>
<tr>
<td>(d) Packers</td>
<td>Cost per person 200 × P14.70 = P2 940.00 – Cost: 5 × P2 940.00 = P14 700.00</td>
<td></td>
</tr>
<tr>
<td>(e) Cleaners</td>
<td>Cost per person P12.70 × 180 = P2 286.00 – Cost: 3 × P2 286.00 = P6 858.00</td>
<td></td>
</tr>
<tr>
<td>Total labour cost</td>
<td>P42 358.00</td>
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</tr>
</tbody>
</table>

2. Materials cost

<table>
<thead>
<tr>
<th>Type of cost</th>
<th>Calculation</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Chicken feed</td>
<td>100 kg × P35.00/kg = P3 500.00</td>
<td></td>
</tr>
<tr>
<td>(b) Incubator costs</td>
<td>P1 570.00</td>
<td></td>
</tr>
<tr>
<td>(c) Nesting material</td>
<td>150 kg × P7.50/kg = P1 245.00</td>
<td></td>
</tr>
<tr>
<td>(d) Newborn chicks</td>
<td>80 × P8.70 = P696.00</td>
<td></td>
</tr>
<tr>
<td>(e) Vaccines</td>
<td>P1 780.00</td>
<td></td>
</tr>
<tr>
<td>(f) Packaging material</td>
<td>P1 460.00</td>
<td></td>
</tr>
<tr>
<td>Total materials cost</td>
<td>P10 131.00</td>
<td></td>
</tr>
</tbody>
</table>

3. Overheads

<table>
<thead>
<tr>
<th>Type of cost</th>
<th>Calculation</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Water</td>
<td>P1 760.00</td>
<td></td>
</tr>
<tr>
<td>(b) Electricity</td>
<td>P2 790.00</td>
<td></td>
</tr>
<tr>
<td>(c) Rent</td>
<td>P3 1340.00</td>
<td></td>
</tr>
<tr>
<td>(d) Telephone</td>
<td>P1 890.00</td>
<td></td>
</tr>
<tr>
<td>(e) Repairs</td>
<td>P5 790.00</td>
<td></td>
</tr>
<tr>
<td>(f) Advertising</td>
<td>P1 630.00</td>
<td></td>
</tr>
<tr>
<td>Total overheads cost</td>
<td>P27 400.00</td>
<td></td>
</tr>
</tbody>
</table>

Subtotal: P79 889.00
V.A.T at 10%: P7 988.90
Total: P87 877.90

(b) Overheads and labour costs are particularly high.
(c) P32 122.10

Exercise 4.2
1. (a) P3.00  (b) P2.60  (c) P16.20
(d) P19.50  (e) P2.30  (f) P15.90
(g) P16.20  (h) P1.10
(i) P28.80  (j) P13.60

2. (a) (i) P3.00  (ii) P3.30  (iii) P3.00  (iv) P4.90  (v) P4.10
(b) P18.30
3. (a) P11.60 (b) P53.90 (c) P40.10  
   (d) P10.80 (e) P139.00  
4. (3 × P16.20) + (2 × P4.10) = P56.80  

**Exercise 4.3**

1. (a) P30.00  
   (b) P103.40  
   (c) P83.30  
   (d) P173.60  
   (e) P99.40  
   (f) P226.00  
2. (a) P111.00  
   (b) P581.20  
   (c) P388.20  
   (d) P171.20  
   (e) P211.60  
   (f) P169.80  
3. Cost by surface mail: P127.50. Cost by airmail: P179.00  
   She will save P51.50 if she mails them by surface mail.

**Activity 4.1**

1. (a) Lesego Kwena  
   (b) Lesego Kwena  
   (c) The bank official  
   (d) 8  
   (e) 145.60  
   (f) One hundred and forty–five pula and sixty thebe  
   (g) To keep track of your deposit.  
   (h) No  

**Exercise 4.4**

1. 

2. (a) (i) Woolworths  
   (ii) Mr P. Masole  
   (iii) P384.30  
   (iv) Three hundred and eighty–four pula and thirty thebe  
   (b) (i) Water Utilities Corporation  
   (ii) Mrs A. Molatole  
   (iii) P846.50  
   (iv) Eight hundred and forty–six pula and fifty thebe  
2. (a) (i) Mrs R. Madome  
   (ii) Pula  
   (iii) Day, month, year  
   (iv) Fifty–seven pula  
   (v) Notes 50: Qty: 1 Amount: 50  
   Coins 5: Qty: 1 Amount: 5  
   Coins 2: Qty: 1 Amount: 2  
   (b) (i) The branch outlet of the bank where the account is held  
   (ii) Mrs R Madome's  
   (iii) Yes  
   (iv) 124.70  
   (v) On the cheque  

**Activity 4.2**

Account name: Student’s name  
Account currency: Pula  
Acc no: The account number  
Date: The date  
Amount withdrawal: P250.00  
Branch: Branch of the bank at which the account is held  
Amount in words: Two hundred and fifty pula  
Customer signature: Student to sign.
**Exercise 4.5**

1. **MONEY ORDER APPLICATION FORM**

   (*FOR CUSTOMER TO COMPLETE IN BLOCK LETTERS*)

<table>
<thead>
<tr>
<th>OFFICE OF PAYMENT:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

   *Names of recipient*  
   1. Mr Richard Mudongo

   *Postal address of recipient*  
   2. 25 Office Towers, Main Street, Gaborone

   *Message*  
   Ref no: TV1087MA

   *Optional services are available at additional charges. (Tick the box if you want the service.)*
   - The Post office to advise the recipient by phone that the money order is waiting for collection.
   - The Post office to provide you with a copy of certified payment.

   *Recipient Cell/Tel*  
   ______________________________

   *Your fax*  
   ________________________  (Placed in your box if no fax)

   *Amount to be paid in words (Pula)*  
   One hundred and twenty-five pula and fifty thebe

   *Name/s of sender*  
   ____________________________________

   *Postal address of sender*  
   ____________________________________

   *Senders Tel/Cell*  
   ______________________________

   *Time the form was handled over*  
   _____________

   **AMOUNT TO BE PAID IN FIGURES**

   - Pula: 125
   - Thebe: 50

   **ISSUING OFFICE USE ONLY**

   **ISSUING OFFICE**

   **ISSUING OFFICE STAMP**

   **Date of issue**

   **Place adhesive Number here**

   **Charges**

   - P
   - t

   **Commission**

   **Fax**

   **Optional**

   **Total**

   **Issuing officer’s signature**

   **Checked by**

   **Mailed/Faxed by**

   **Date**

   **Time**

   **NOTE: (FOR USE BY PAYING OFFICE ONLY)**

   **(Individual)**

   **Name of recipient**

   **Signature of recipient**

   **Identity number**

   **(Company)**

   **Names of representative collecting**

   **Signature of Representative**

   **Identity Number**

   **Names of paying officer**

   **Signature of paying officer**

   **PAYING OFFICE STAMP**

   **Recipient phoned: date**

   **Time**

   **Recipient advised: date**

   **Time**

   **Sender faxed: date**

   **Time**

   **Faxed Office Copy**
2. You are guaranteed the money, since the money order has already been paid for.
3. No cash is sent so the money cannot be stolen.
4. 
   P38.40
   P350 + P38.40 = P388.40
5. P34.50

Activity 4.3

1. Mrs K. Moswela  2. China
3. Mr R. Lee   4. US$560.00
5. P3 808.00   6. Barclays Bank
7. To ensure that the writing is legible.

Activity 4.4

Exercise 4.6

1. 

<table>
<thead>
<tr>
<th>Type of loan</th>
<th>Explanation</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Variable home loan</td>
<td>The interest rate moves up or down in accordance with movements in global interest rates.</td>
<td>Repayments fall when interest rates fall.</td>
<td>Repayments rise when interest rates rise.</td>
</tr>
<tr>
<td>2. Fixed rate home loan</td>
<td>The interest rate is fixed for a certain period of the home loan period.</td>
<td>You are always assured of how much you will pay each month.</td>
<td>If interest rates fall, you do not benefit in any way.</td>
</tr>
<tr>
<td>3. Interest–only home loan</td>
<td>You repay only the interest on the principal for 1 – 5 years. Thereafter, you must start making principal and interest repayments.</td>
<td>Initially, you pay much less which is good if you are short of cash.</td>
<td>There will be a sudden increase in repayments at the end of the interest–only period.</td>
</tr>
<tr>
<td>4. Introductory home loan</td>
<td>An initial low interest rate is charged in order to attract borrowers. This rate usually only lasts for around 12 months and then reverts to the standard interest rate.</td>
<td>Payments in the low-interest period can reduce the principal quickly.</td>
<td>There will be a sudden increase in repayments when the low interest rate period is over.</td>
</tr>
<tr>
<td>5. No deposit home loan</td>
<td>No deposit is required.</td>
<td>Perfect for borrowers that do not have enough money for a deposit.</td>
<td>Higher interest rate than traditional loans.</td>
</tr>
</tbody>
</table>
2. A secured loan is a loan in which the borrower promises some asset, such as a car or property, as security for the loan. Example: A mortgage loan where the house is the asset. An unsecured loan is not secured against any assets. Example: A personal loan.

3. This depends on whether or not Mrs Molefe can afford the 10% deposit. If she can pay the deposit, she will pay P1 350.00 in interest per year. For the second option, she will pay P1 650.00 interest per year. So option 1 is best if she can afford the deposit.

4. This will all depend on whether or not Kwena is sure that she can make her repayments.
   Option 1: A higher interest rate, but this is a better option if she thinks she might not be able to make a payment, as she then won’t lose her car.
   Option 2: A lower interest rate, but she must be sure that she can maintain her payments as her car may be repossessed if she does not pay in time.

Activity 4.6
1. Microwave: P1 81.00
   Fridge: P390.00
   Sleeper couch: P8 400.00
   Television: P2 91.00
   Vacuum cleaner: P2 41.00

2. HP is perfect for people who do not have enough money to buy the item up front. The big disadvantage is the much larger overall total cost using the HP option.

Exercise 4.10
1. (a) P86.87  (b) P108.13  (c) P92.49  (d) P113.00  (e) P116.94
2. (a) P117.35  (b) P694.19  (c) P120.35  (d) P671.69  (e) P121.90  (f) P124.95  (g) P659.13  (h) P655.38  (i) P135.88  (j) P649.00
3. (a) P694.75  (b) P8 177.25  (c) P98 127.00

4. P102.88 – P90.24 = P12.64
5. P89.37 + P89.37 + P112.44 + P98.74 = P387.54
6. P1 938.44

Activity 4.7
2. (a) P0  (b) P0  (c) P750.00
   (d) P1 500.00 + P2 500.00 = P1 750.00
   (e) P5 250.00 + P4 230.00 = P9 480.00
   (f) P5 250.00 + P980.63 = P6 230.63
   (g) P10 875.00 + P8 825.00 = P19 700.00
   (h) P10 875.00 + P2 750.00 = P13 625.00

Exercise 4.11
1. (a) P0  (b) P0  (c) P250.00
   (d) P1 500.00 + P2 050.00 = P3 550.00
   (e) P5 250.00 + P4 230.00 = P9 480.00
   (f) P5 250.00 + P980.63 = P6 230.63
   (g) P10 875.00 + P8 825.00 = P19 700.00
   (h) P10 875.00 + P2 750.00 = P38 625.00

2. (a) P10 875.00 + P450.00 = P11 325.00
   (b) P110 475.00
   (c) Yes, he will be able to save P706.25 per month.
3. (a) P5 250.00 + P3 093.75 = P8 343.75 (b) P695.31 (c) P8 179.69

4. Mrs Moswela: Annual tax is P1 625.00 Mr Koloi: Annual tax is P1 475.00 Mrs Moswela: Net monthly income: P4 947.92 Mr Koloi: Net monthly income: P4 835.42 False. Mrs Moswela takes home more money per month.

5. (a) P1 875.00 (b) P3 375.00

6. Botswana residents pay no tax on the first P30 000 of earnings, whereas a non–resident is charged 5% tax on all money earned up to P60 000.

7. (a) P3 500.00 (b) P175.00 (c) P3 325.00

**Exercise 4.12**
Answers are all to 2 decimal places.

1. 

<table>
<thead>
<tr>
<th>Item</th>
<th>Price excl. sales tax</th>
<th>Price incl. sales tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Frothy cappuccino</td>
<td>P6.50</td>
<td>P7.02 ≈ P7.00</td>
</tr>
<tr>
<td>3. Creamy chicken casserole</td>
<td>P39.90</td>
<td>P43.092 ≈ P43.10</td>
</tr>
<tr>
<td>4. Tropical salad with apple and cashews</td>
<td>P17.80</td>
<td>P19.224 ≈ P19.20</td>
</tr>
<tr>
<td>5. Rich caramel surprise</td>
<td>P20.70</td>
<td>P22.356 ≈ P22.40</td>
</tr>
</tbody>
</table>

2. (a) P210.60 (b) P198.00 with VAT, so P12.60 less

3. Tax is P170.10, so total bill is P2 060.10.

4. (a) P411.80 (b) P176.50

5. (a) P359.10 (b) P39.00 – P34.10 = P4.90 more

6. (a) P5 716.48 = P5 716.50 (b) P5 716.48 = P5 716.50 (c) Answers are the same.

7. **Challenge:** P13 750.00

**Exercise 4.13**

1. (a) P2 050.00 (b) P1 615.00 (c) P8 400.00 (d) P15 525.00

2. (a) P1 585.00 (b) P1 350.00 (c) P18 785.00

3. P20 295.00

4. (a) P4 860.00 (b) P3 375.00 (c) P56 835.00

5. (a) P200 200.00 (b) P4 104.00

**Exercise 4.14**

1. P70.00 2. P240.00

3. (a) P70.00 (b) P140.00 (c) P90.00 (d) P110.00 (e) P190.00 (f) P60.00 (g) P160.00 (h) P260.00

4. (a) P190.00 (b) P6 700.00 (c) P69 000.00

5. To keep track of ownership and to find stolen cars.

6. 4 500 kg

7. Cost of car: P43 250.00; VAT (10%): P4 325.00; registration: P90.00. Total needed: P47 665.00. Her uncle is incorrect. She needs at least P47 665.00 to buy the vehicle.

**Activity 4.8**
The advantage is that if you sell a lot of merchandise, you will get more money. This motivates sales people to work harder. The disadvantage is that if you don’t sell, no matter how hard you work, you will get no pay.

**Exercise 4.15**

1. (a) Thapelo: P961.20 (b) Atlang: P166.00 (c) Julia: P797.20

2. P37 800.00

3. Job 1: P348.00 for the week. Job 2: P255.00 plus commission.

If Joe wants a secure income, then job 1 is better. If he thinks the television business is a good one and he is sure that he can fix many televisions, then job 2 might be a better option, as he could earn good commission.

4. P20 880.00 5. P6 200.00

6. **Hungry Jacks – Commission record**

<table>
<thead>
<tr>
<th>Employee</th>
<th>Commission rate (% of the value of the meals sold)</th>
<th>Total value of the meals sold for the week</th>
<th>Money made in commission</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Ngwakwana</td>
<td>5%</td>
<td>P1 250</td>
<td>P62.50</td>
</tr>
<tr>
<td>(b) John</td>
<td>7.5%</td>
<td>P980</td>
<td>P73.50</td>
</tr>
<tr>
<td>(c) Boitshepo</td>
<td>6%</td>
<td>P1 000</td>
<td>P60.00</td>
</tr>
<tr>
<td>(d) Lesego</td>
<td>8%</td>
<td>P980</td>
<td>P68.00</td>
</tr>
<tr>
<td>(e) Kitso</td>
<td>6.5%</td>
<td>P1 300</td>
<td>P84.50</td>
</tr>
</tbody>
</table>

Kitso is the star employee of the month.

7. **Challenge:** P13 250.00
Revision exercise

1. Water, electricity, rent, telephone, advertising, etc.
2. P267.20
3. Payee, date of cheque, amount in figures, amount in words, signature.
4. A cheque draws money from your own cheque account. If you have no money in your cheque account, the cheque will “bounce” and not be payable. A money order is purchased from a post office or other institution. It is paid for in cash, so the recipient is assured of getting the money.
5. Sending foreign currency abroad.
6. P250.00
7. P8 500.00
8. P1 350.00
9. P4 477.00
10. (a) P5 812.50  (b) P484.38
11. P492.75
12. P8 720.00
13. P46 820.00
14. P869.00
Chapter 5 Matrices

Exercise 5.1
1. (a) 3 rows; 3 columns (b) 2 rows; 2 columns (c) 3 rows; 2 columns (d) 3 rows; 5 columns
2. (a) \[
\begin{pmatrix}
10 & 12 & 8 \\
15 & 10 & 12 \\
14 & 8 & 12
\end{pmatrix}
\] (b) 3rd row; 1st column (c) 2nd row; 3rd column
3. (a) \[
\begin{pmatrix}
50 & 120 & 300 \\
80 & 180 & 120 \\
120 & 250 & 300 \\
200 & 250 & 100 \\
250 & 200 & 150 \\
120 & 120 & 120
\end{pmatrix}
\] (b) Day 4 (c) Day 3 (d) 2nd row; 3rd column
4. (a) 2nd row (b) 1st column (c) First Sunday: 65 Second Sunday: 88 Difference: 23 people
5. (a) \[
\begin{pmatrix}
5 & –2 & 2 \\
0 & 4 & –1 \\
8 & 5 & –1
\end{pmatrix}
\] (b) 2nd April (c) Lift 1 (d) You could be trapped in the lift, as the lift may stop working. It is always best to use the stairs as an exit point when leaving a building that is on fire.

Activity 5.1
1 and 2 – Answers will vary.
3. A matrix is less bulky than a table, showing only the essential numbers, rather than the headings and the lines drawn in to differentiate between each row and column. Matrices can be added, subtracted, multiplied and divided, whereas information in a table would need to be transferred out of the table before any operations could be performed on the data. A table, however, is perhaps easier to use if the data is to be analysed as is. This is because a table has headings that explain the data in words.

Exercise 5.2
1. (a) 1 × 4 (b) 3 × 1 (c) 3 × 3 (d) 3 × 5 (e) 2 × 3 (f) 4 × 4
2. (c) and (f)
3. (a) 3 × 4 (b) 2 × 2 (c) 4 (d) –3 (e) 3rd row; 4th column
4. (a) Any matrix with 3 rows and 2 columns. (b) Any matrix with 2 rows and 2 columns. (c) \[
\begin{pmatrix}
8 & 16 & 24 \\
0 & 0 & 0 \\
4 & 8 & 12
\end{pmatrix}
\]
5. False. The order is 2 × 4.
6. (a) 4 × 5 (b) 13 × 10 (c) 6 × 7 (d) 8 × 1
7. (a) \[a_{11} = 1; a_{12} = 4; a_{13} = 5; a_{21} = 8; a_{22} = 2; a_{23} = 9\] (b) \[b_{11}; b_{22}; b_{33}; b_{44}; b_{55}; b_{66}; b_{77}; b_{88}\]

Activity 5.2
1. A: 2 × 2 B: 1 × 3 C: 2 × 2 D: 4 × 1
2. No. There are not enough rows in A to match with the number of rows in D.
3. No. There are not enough rows in C to match with the number of rows in D.
4. Yes. There are the same number of rows and columns in both A and C.

Exercise 5.3
1. (a) \[
\begin{pmatrix}
7 & 4 \\
0 & 3
\end{pmatrix}
\] (b) \[
\begin{pmatrix}
1 & –1 \\
–1 & 3
\end{pmatrix}
\] (c) \[
\begin{pmatrix}
11 & 1 \\
–1 & –1
\end{pmatrix}
\] (d) \[
\begin{pmatrix}
5 & 5 & 2 & –1 \\
2 & 3 & 1 & 8 \\
–2 & 9 & –5 & 0
\end{pmatrix}
\]
(e) \[
\begin{pmatrix}
12 & –10 & 13 \\
14 & 15 & –7 \\
1 & –1 & 11
\end{pmatrix}
\] (f) \[
\begin{pmatrix}
0 & 13 & 8
\end{pmatrix}
\]
2. (a) \[
\begin{pmatrix}
11 & 5 & 1 \\
0 & 1 & 3
\end{pmatrix}
\] Order is 2 × 3.
(b) \[
\begin{pmatrix}
2 & –1 \\
4 & 5 \\
8 & 4
\end{pmatrix}
\] Order is 3 × 2.
(c) Matrices A and B and matrices C and D do not have the same number of rows and columns.
3. (a) Answers will vary, but here is one correct answer:
\[
A = \begin{pmatrix}
3 & 5 & 7 \\
5 & 3 & 7
\end{pmatrix}
\]
\[
B = \begin{pmatrix}
–4 & –2 & –6 \\
–6 & –4 & –2
\end{pmatrix}
\] (b) Answers will vary, but using the matrices from (a): \[A + B = \begin{pmatrix}
–1 & 3 & 1 \\
–1 & –1 & 5
\end{pmatrix}\]
4. (a) \[X + Y = \begin{pmatrix}
15 & 3 & –6 \\
10 & 3 & 11
\end{pmatrix}\] (b) Addition is not possible.
(c) \[E + F = \begin{pmatrix}
1 & 2 & –2 \\
9 & 8 & 2
\end{pmatrix}\]
(d) \[B + C = \begin{pmatrix}
12 & 11 \\
–4 & –1 \\
29 & –18
\end{pmatrix}\] (e) Addition is not possible.
5. \(6 + x = 2\), so \(x = –4\) \(w + 0 = 8\), so \(w = 8\)
\(2 + y = –5\), so \(y = –5\) \(–3 + 7 = z\), so \(z = 4\)
Exercise 5.4
1. (a) \( \begin{pmatrix} -2 & -2 \\ -8 & 5 \end{pmatrix} \)
   (b) \( \begin{pmatrix} -5 & 2 & 2 \\ 1 & -4 & -2 \end{pmatrix} \)
   (c) Matrices F and H do not have the same number of rows and columns.
2. (a) \( \begin{pmatrix} 2 & -1 \\ -2 & -1 \end{pmatrix} \)
   (b) \( \begin{pmatrix} 3 & -1 & -2 \\ -5 & 2 & -7 \\ 1 & -2 & 6 \end{pmatrix} \)
   (c) \( -2, 5, 2, -9 \)
   (d) \( \begin{pmatrix} 1 & 0 & 3 \\ -2 & 2 & 5 \\ -5 & -2 & -4 \end{pmatrix} \)
   (e) \( -2, -1 \)
   (f) \( \begin{pmatrix} 2 & 0 & -1 \\ 4 & 1 & 0 \\ 0 & 1 & 2 \\ 3 & 5 & 4 \end{pmatrix} \)
3. (a) \( N = \begin{pmatrix} -1 & -1 \\ -1 & -1 \\ -1 & -1 \\ -1 & -1 \\ -1 & -1 \end{pmatrix} \)
   (b) \( O = \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \\ 4 & 4 & 4 \end{pmatrix} \)
   (c) \( O - N = \begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{pmatrix} \)
4. (a) \( \begin{pmatrix} -7 & -6 & 1 \\ 4 & 0 & -2 \end{pmatrix} \)
   (b) \( \begin{pmatrix} 4 & 1 & -5 \\ 4 & -3 & -1 \\ -6 & 8 & -2 \end{pmatrix} \)
   (c) \( \begin{pmatrix} -3 & 0 & -3 \\ 2 & 2 & 2 \end{pmatrix} \)
   (d) \( \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 5 \\ 7 & 1 & 2 \end{pmatrix} \)
5. (a) \( x - 1 = -3, \) so \( x = -2 \)
   (b) \( x = 0, \) so \( y = -3 \)
   (c) \( 2x - 4 = 2 \)
   \( \therefore 2x = 6 \)
   \( \therefore x = 3 \)
   (d) \( 3 - y = -1, \) so \( y = 4 \)
   (e) \( 5 \) - \( y + 1 = 0 \)
   \( \therefore 5 - y = 0 \)
   \( \therefore y = 4 \)

Exercise 5.5
1. (a) \( \begin{pmatrix} 5 & 0 \\ -10 & 15 \end{pmatrix} \)
   (b) \( \begin{pmatrix} -6 & 0 & 2 & -12 \end{pmatrix} \)
   (c) \( \begin{pmatrix} 4 & 12 & 12 \\ 8 & 20 & 12 \\ -16 & 16 & 0 \end{pmatrix} \)
   (d) \( \begin{pmatrix} 21/2 \\ 1/2 \end{pmatrix} \)
   (e) \( \begin{pmatrix} 72 & -24 & 36 & 48 \\ -48 & 12 & 0 & 60 \end{pmatrix} \)
   (f) \( \begin{pmatrix} 15p & -6p \\ 12p & 9p \end{pmatrix} \)
2. (a) \( \begin{pmatrix} 8 & 4 \\ 4 & 6 \end{pmatrix} \)
   (b) \( \begin{pmatrix} -8 & 0 \\ 0 & 16 \end{pmatrix} \)
   (c) \( \begin{pmatrix} -5 & 2 \\ -3 & -1 \end{pmatrix} \)
   (d) \( \begin{pmatrix} 15 & 6 \\ 9 & 3 \end{pmatrix} + \begin{pmatrix} -4 & 0 \\ 8 & 10 \end{pmatrix} = \begin{pmatrix} 11 & 6 \\ 17 & 13 \end{pmatrix} \)
   (e) \( \begin{pmatrix} -4 & -2 \\ -3 & 2 \end{pmatrix} \)
   (f) \( \begin{pmatrix} 20 & 8 \\ 12 & 4 \end{pmatrix} - \begin{pmatrix} -6 & 0 \\ 12 & 15 \end{pmatrix} = \begin{pmatrix} 26 & 8 \\ 0 & -11 \end{pmatrix} \)
3. (a) \( \begin{pmatrix} 1 & 2 & 3 \\ 5 & 10 & 8 \\ 3 & 2 & 4 \end{pmatrix} \)
   (b) Bucket in the shower to catch excess water.
   (c) \( \begin{pmatrix} 5 & 10 & 15 \\ 25 & 50 & 40 \end{pmatrix} \)
   (d) \( \begin{pmatrix} 15 & 10 & 20 \end{pmatrix} \)
4. (a) \( \begin{pmatrix} -10 & -6 & -4 \\ 8 & -10 & 4 \\ 0 & 2 & -6 \end{pmatrix} \)
   (b) \( \begin{pmatrix} -4 & 0 \\ -6 & 2 \end{pmatrix} \)
5. \( Y = \begin{pmatrix} 3 & 4 & 7 \\ 10 & 12 & 9 \\ 2 & 1 & 5 \end{pmatrix} \)
6. \( 8x = -16, \) so \( x = -2 \)
   \( 8y = -8, \) so \( y = -1 \)
   \( 8z = 96, \) so \( z = 12 \)
7. (a) \( \begin{pmatrix} 23.25 & 15.50 & 12.40 \\ 31.00 & 18.60 & 15.50 \\ 40.30 & 38.75 & 31.00 \end{pmatrix} \)
   (b) \( \begin{pmatrix} 232.50 & 155.00 & 124.00 \\ 310.00 & 186.00 & 155.00 \\ 403.00 & 387.50 & 310.00 \end{pmatrix} \)
   (c) \( \begin{pmatrix} 2 & 63 \end{pmatrix} \)

Exercise 5.6
1. (a) 11  (b) -5  (c) 70
   (d) Not possible
2. \( x = -2 \)
3. (a) \( \begin{pmatrix} 12 & 25 & 32 \end{pmatrix} \)
   (b) \( \begin{pmatrix} 4.00 & 3.50 & 6.00 \end{pmatrix} \)
   (c) \( P = 339.50 \)
4. (a) 24  (b) 5
   (c) The number of columns of matrix W is not equal to the number of rows of matrix Y.

Activity 5.3
1. (16 22)
2. (–14 –6)
3. (–26 –26)
4. (8 32)

Exercise 5.7
1. (a) \( \begin{pmatrix} 2 \end{pmatrix} \)
   (b) \( \begin{pmatrix} 6 & 32 \end{pmatrix} \)
   (c) \( \begin{pmatrix} -14 & -8 \end{pmatrix} \)
   (d) \( \begin{pmatrix} -1 \end{pmatrix} \)
   (e) \( \begin{pmatrix} 17 & -7 \end{pmatrix} \)
   (f) \( \begin{pmatrix} -26 \end{pmatrix} \)
2. (a) \( \begin{pmatrix} 9 & 1 \\ -3 & -4 \end{pmatrix} \)
   (b) \( \begin{pmatrix} 16 & 2 \\ -1 & -2 \end{pmatrix} \)
   (c) \( \begin{pmatrix} 9 & -2 \\ 19 & 8 \end{pmatrix} \)
   (d) \( \begin{pmatrix} 3 & 0 \\ -6 & -5 \end{pmatrix} \)
   (e) \( \begin{pmatrix} 9 & 2 \\ 0 & 1 \end{pmatrix} \)
   (f) \( \begin{pmatrix} 1 & -6 \\ 18 & 13 \end{pmatrix} \)
   (g) \( \begin{pmatrix} -1 & 4 \\ -8 & 7 \end{pmatrix} \)
   (h) \( \begin{pmatrix} 2 & 3/2 \end{pmatrix} \)
3. (a) \( m - 10 = -7 \), so \( m = 3 \)
   \( n = 18 \)
(b) \( 0 + 2m = 6 \), so \( m = 3 \)
   \( 1 + 3 = n \), so \( n = 4 \)

4. (a) (i) \[
\begin{pmatrix}
8 & -8 \\
31 & -20
\end{pmatrix}
\] (ii) \[
\begin{pmatrix}
2 & -1 \\
5 & 3
\end{pmatrix}
\] (iii) \[
\begin{pmatrix}
5 & -4 \\
2 & 0
\end{pmatrix}
\]
(b) The original matrix stays the same.
(c) Answers will vary, but learners will see that multiplying by \[
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\]
will leave them with the same matrix that they started with.

Revision exercise

1. (a) 2 × 4   (b) 2 × 2   (c) 3 × 3   (d) 1 × 4
2. (b) and (c)
3. (a) \[
\begin{pmatrix}
8 & 2 & 2 \\
2 & 5 & 3
\end{pmatrix}
\]
   (b) \[
\begin{pmatrix}
1 & 1 \\
1 & -3
\end{pmatrix}
\]
   (c) \[
\begin{pmatrix}
1 & 0 & 1 \\
8 & 4 & 3 \\
0 & -1 & 8
\end{pmatrix}
\]
   (d) \[
\begin{pmatrix}
1 & -2 \\
-1 & -4 \\
-8 & 0
\end{pmatrix}
\]
4. (a) \[
\begin{pmatrix}
25 & 20 \\
1.25 & 2.50
\end{pmatrix}
\]
   (b) \[
\begin{pmatrix}
300 & 240 \\
15 & 30
\end{pmatrix}
\]
   Clown Around Circus will cost P315 and Whizz Bang Circus will cost P270. It will be cheaper to go to Whizz Bang Circus.
5. (a) 12   (b) Not possible
   (c) Not possible   (d) \[
\begin{pmatrix}
17 & -6 \\
-3 & 2
\end{pmatrix}
\]
   (e) \[
\begin{pmatrix}
17 & 6 \\
3 & 2
\end{pmatrix}
\]
   (f) \[
\begin{pmatrix}
30 & 12 \\
-6 & 0
\end{pmatrix}
\]
   (g) \[
\begin{pmatrix}
1 & 1 \\
2 & -1
\end{pmatrix}
\]
   (h) Not possible
   (i) Not possible   (j) \[
\begin{pmatrix}
34 & -12 \\
-6 & 4
\end{pmatrix}
\]
Chapter 6 Angle properties

1. 75º

2. (a) 75º (b) 51º
   (c) 30º (d) 51º
   (e) b = 50º; c = 50º
     a = 49º; c = 68º (f) b = 63º;

3. (a) 79º (b) 66º (c) 53º
   (d) 36º (e) 45º

4. (a) 3 = 87º; 2 = 93º; 1 = 59º
   (b) 3 = 92º; 2 = 88º; 1 = 65º

5. (a) x + 2x + 4x = 180º
   \[
   \begin{align*}
   \therefore 9x & = 180º \\
   \therefore x & = 20º \\
   e & = 40º; f = 60º; g = 80º
   \end{align*}
   
   6. Challenge:

   2x + 3x + 4x = 180º

   \[
   \begin{align*}
   \therefore 9x & = 180º \\
   \therefore x & = 20º \\
   e & = 40º; f = 60º; g = 80º
   \end{align*}
   
   Exercise 6.2

   1. 54º
   2. PQ = PR
   3. (a) b = c = 62º (b) c = 35º; b = 110º
      (c) b = c = 48º; a = 84º (d) b = c = 65º
   4. E = 45º
   5. If 80º is the vertex angle, then the base angles will be 50º each. If 80º is one of the base angles, then the other base angle will be 80º and the vertex angle will be 20º.

   Activity 6.1

   Let the triangle be triangle ABC. AB = AC. We know that angle B = angle C since the triangle is isosceles. If angle B is greater than or equal to 90º, then angle C is greater than or equal to 90º. Let us assume that angles B and C are equal to 90º. If B = C = 90º, then angle B + angle C = 180º. But the sum of the three interior angles of a triangle is 180º so that leaves 0º for angle A. Angle A cannot be 0º or else there will be no vertex at A. Therefore, angles B and C cannot be equal to 90º. It follows that if angle B and angle C are greater than 90º, then their sum will be greater than 180º. We cannot have this since the interior angles of any triangle always add up to 180º and no more. Therefore, angles B and C must be acute (less than 90º).

   Activity 6.2

   Triangle MNO is an equilateral triangle which means that angle M = angle N = angle O. Let each angle be equal to x. Since the sum of the interior angles of a triangle is 180º, x + x + x = 180º. So, 3x = 180º and x = 60º. Therefore, every angle of an equilateral triangle is equal to 60º.

   Exercise 6.3

   1. (a) g = h = i = 60º
   2.

   3. (a) a = b = c = 60º
      (b) x = 60º therefore a = b = c = 60º
      (c) a = b = c = 60º (d) x = y = z = 60º
   4. Yes DEF is an equilateral triangle.

   Activity 6.3

   Investigation:

   It does not matter where the house is built. The sum of the three driveway lengths will always be the same. This only works for an equilateral triangle, not other triangles. Pick any point P inside an equilateral triangle. The sum of the perpendicular distances from any point P inside the triangle to the three sides of the triangle is always the same.
Activity 6.4
Proving that \( r = m + n \):
\[ m + n + o = 180^\circ \] (interior angles of a triangle)
But \( o + r = 180^\circ \) (adjacent angles on a straight line)
\[ o + r = m + n + o \]
\[ \therefore r = m + n + o - o \]
\[ = m + n \]
Proving that \( p = n + o \):
\[ m + n + o = 180^\circ \] (interior angles of a triangle)
But \( p + m = 180^\circ \) (adjacent angles on a straight line)
\[ p + m = m + n + o \]
\[ \therefore p = m + n + o - m \]
\[ = n + o \]

Exercise 6.4
1. (a) \( x = 113^\circ \)   (b) \( x = 55^\circ \)
   (c) \( x = 130^\circ \)   (d) \( x = 60^\circ \)
2. (a) Exterior angle at N = 120º
    (b) \( l = 50^\circ; m = 70^\circ \) and \( n = 60^\circ \)
3. (a) Exterior angle at Y = 90º and at W = 120º
    (b)

4. Exterior angle at K = 150º;
   exterior angle at L = 105º;
   exterior angle at M = 105º
5. 120º

Activity 6.5
Investigation:
A complete revolution is 360º. Students will see that as they walk around the triangle, they are turning their bodies at each exterior angle. When they end, they are left facing where they started. They have walked a complete revolution. This helps to understand that the sum of the exterior angles of a triangle is 360º.

Exercise 6.5
(a) \( x = 60^\circ; y = 120^\circ \)   (b) \( y = 95^\circ; x = 37^\circ \)
(c) \( x = 57^\circ; y = 123^\circ \)
(d) \( y = 77^\circ; x = 26^\circ; z = 103^\circ \)
(e) \( x = 91^\circ; y = 55^\circ; z = 34^\circ \)
(f) \( 4x + 10^\circ = 180^\circ \)
\[ \therefore 4x = 170^\circ \]
\[ \therefore x = 42.5^\circ, \text{ so angle } a = 52.5^\circ, b = 42.5^\circ \text{ and } c = 85^\circ \]

(g) \( x = 60^\circ; y = z = 30^\circ \)
(h) \( x = y = 77^\circ; z = 154^\circ \)
(i) \( x = 115^\circ; y = 125^\circ; z = 120^\circ \)
(j) \( x = y = 45^\circ; z = 135^\circ \)

Exercise 6.6
1. (a) All angles are equal to 90º.
   (b) Opposite angles are equal. Diagonals bisect at 90º.
   (c) One pair of opposite angles are equal. Diagonals intersect at 90º.
   (d) All angles are equal to 90º. Diagonals bisect at 90º.
   (e) No angles are necessarily equal. For a right–angled trapezium, one angle is equal to 90º. For an isosceles trapezium, base angles are equal.
   (f) Opposite angles are equal.
2. (a) 68º   (b) 60º   (c) 132º   (d) 70º
   (e) 141º   (f) 45º
3. (a) \( a = b = c = d = 90^\circ \)
   (b) \( q = 43^\circ; p = r = 137^\circ \)
   (c) \( k = m = 90^\circ \)
   (d) \( x = 120^\circ; y = 40^\circ; w = 80^\circ \)
   (e) \( e = g = 110^\circ; f = h = 70^\circ \)
   (f) \( v = 45^\circ \) (alternate angles); \( w = 135^\circ; t = 90^\circ \)
   (g) \( d = b = 96^\circ; a = c = 84^\circ \)
   (h) \( a = 70^\circ \) (corresponding angles); \( m = 128^\circ \)
4. (a) \( k = 75^\circ; i = 102^\circ \)
   (b) \( x = 80^\circ; u = 105^\circ; v = 95^\circ \)
5. 78º
6. The two smaller angles are equal to 65º and the two larger angles are equal to 115º.
7. The two missing angles are both equal to 135.5º.
8. (a) \( x = 30^\circ \) and \( 2x = 60^\circ \)
   (b) \( a = 74^\circ; c = 44^\circ; b = 118^\circ \)
9. Yes, if we know one angle, then we know two angles, since opposite angles are equal. Once we know two angles, we can subtract their sum from 360º to find the sum of the two remaining angles. Since the two remaining angles are equal, we can divide their sum by 2 to find each individual angle.
10. The two smaller angles are equal to 83°. The two larger angles are equal to 97°.

**Revision exercise**

1. (a) \( x = 73^\circ \)  
   (b) \( x = 50^\circ \)  
   (c) \( x = y = 67.5^\circ \)  
   (d) \( x = 30^\circ; y = 121^\circ \)  
   (e) \( x = y = z = 60^\circ \)  
   (f) \( x = 88^\circ; y = 115^\circ; z = 153^\circ \)

2. (a) \( x = 40^\circ; y = 130^\circ \)  
   (b) \( x = 65^\circ; y = z = 115^\circ \)  
   (c) \( x = 96^\circ \)  
   (d) \( x = 120^\circ; y = 65^\circ \)  
   (e) \( x = 63^\circ; y = 104^\circ \)  
   (f) \( x = 49^\circ; y = 90^\circ; z = 131^\circ \)